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Constraint on the heavy sterile neutrino mixing angles in the SO(10) model with double see-saw mechanism

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Abstract. Constraints on the heavy sterile neutrino mixing angles are studied in the framework of a minimal supersymmetric SO(10) model with the use of the double see-saw mechanism. A new singlet matter in addition to the right-handed neutrinos is introduced to realize the double see-saw mechanism. The light Majorana neutrino mass matrix is, in general, given by a combination of those of the singlet neutrinos and the active neutrinos. The minimal SO(10) model is used to give an example form of the Dirac neutrino mass matrix, which enables us to predict the masses and the mixing angles in the enlarged 9×9 neutrino mass matrix. Mixing angles between the light Majorana neutrinos and the heavy sterile neutrinos are shown to be within the LEP experimental bound on all ranges of the Majorana phases.

1 Introduction

Recent neutrino oscillation data opened up a new window to prove physics beyond the standard model. As pointed out in [1], we can construct, within the context of the standard model (SM), an operator which gives rise to the neutrino masses as

$$\mathcal{L}_{\text{eff}} = \frac{1}{\Lambda}(\ell_L H)^{\mathrm{T}} C^{-1}(\ell_L H)\,. \tag{1}$$

Here ℓ, H are the lepton doublet and the Higgs doublet, C is the charge conjugation operator and Λ is the scale in which something new physics appears. This operator can naturally be arisen in the see-saw mechanism [2–4], which may give a guideline to construct models of new physics through the existence of the right-handed neutrinos.

On the other hand, the supersymmetric (SUSY) grand unified theory (GUT) provides an attractive implication for the understandings of the low-energy physics. In fact, for instance, the anomaly cancellation between the several matter multiplets is automatic in the GUT based on a simple gauge group, since the matter multiplets are unified into a few multiplets, the experimental data supports the fact of unification of three gauge couplings at the GUT scale $M_{\mathrm{GUT}} = 2 \times 10^{16}$ GeV assuming the particle contents of the minimal supersymmetric standard model (MSSM), and also the right-handed neutrino appeared naturally in the SO(10) GUT provides a natural explanation of the smallness of the neutrino masses through the see-saw mechanism [2–4].

Although the essential concept of the see-saw mechanism is the same, there can be many possibilities according to the types of the see-saw mechanism. For instance, as motivated by the superstring inspired E_6 models, we come to consider *the double see-saw mechanism* [5–9] and it's extension, the type-III see-saw mechanism [10] (see also, [11, 12]). Interestingly, in such an extension of the standard see-saw mechanism, it may appear the singlet neutrinos in the reachable range of the future collider experiments. The possibility of testing the not-so-heavy singlet neutrinos at collider experiments has firstly been proposed by [13] and subsequently analyzed by the LEP collaborations [14].

In this letter we give constraints on the mixing angles between active and sterile neutrinos in the enlarged 9×9 mass matrix which appears in the double see-saw mechanism using an SO(10) model with double see-saw mechanism. The constraints on the mixing angles are imposed so as to satisfy the current neutrino oscillation data.

We accept the same Lagrangian as in [8]. That is, we add a new singlet matter (S) in addition to the right-handed neutrino (ν^c) per a generation. The Lagrangian in this model is given by

$$\mathcal{L}_Y = Y_\nu^{ij} \nu_i^c L_j H_u + Y_s^{ij} \nu_i^c S_j H_s + \mu_s S_i^2 + \text{h.c.}\,, \tag{2}$$

where L_j is the lepton doublet, and H_u, H_s are the $\mathrm{SU}(2)_L$ doublet, singlet Higgs fields.

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Note that the μ_s term in the above breaks an originally existing global $\mathrm{U}(1)_L$ (lepton number) (and $\mathrm{U}(1)_{\mathcal{R}}$ symmetry in the case of supersymmetry). Thus we can naturally expect it as a small value compared with the electroweak scale even around the keV scale, according to the following reason: when the μ_s term is arisen from the VEV of a singlet $\mu_s = \lambda\langle S'\rangle$, there appears a pseudo-NG boson, called Majoron $J = \mathrm{Im}\, S'$ associated with the spontaneously broken $\mathrm{U}(1)_L$ symmetry. Then the keV scale lepton number violation may lead to an interesting signature in the neutrinoless double beta decay [15] or becomes a possible candidate for the cold dark matter [16].

The mass terms of the Lagrangian (2) are re-written in a matrix form in the base with $\{\nu, \nu^c, S\}$ as follows [5–9],

$$\mathcal{M} = \begin{pmatrix} 0 & m_D & 0 \\ m_D^{\mathrm{T}} & 0 & M_D \\ 0 & M_D^{\mathrm{T}} & \mu_s \mathbb{I} \end{pmatrix} . \tag{3}$$

Here $m_D \equiv Y_\nu \langle H_u\rangle$, $M_D \equiv Y_s\langle H_s\rangle$, and $\mathbb{I} \equiv \mathrm{diag}(1,1,1)$. In this paper we assume that the mass matrix M_D is written in terms of a unitary matrix V as $(M_D)_{ij} = V_{ij}^* M_{Dj}$, where the unitary matrix V diagonalises a combination $M_D M_D^{\mathrm{T}}$,

$$V^{\mathrm{T}} M_D M_D^{\mathrm{T}} V = \mathrm{diag}\left(M_{D1}^2, M_{D2}^2, M_{D3}^2\right) . \tag{4}$$

On the other hand, the full 9×9 mass matrix (3) can be diagonalised by using a unitary matrix U as

$$U^{\mathrm{T}}\mathcal{M}U = \mathrm{diag}(m_1, m_2, m_3, \underbrace{m_{N_1}, m_{N_2}, \ldots, m_{N_5}, m_{N_6}}_{\text{heavy isosinglet neutrinos}}) , \tag{5}$$

where $m_{N_1} \simeq m_{N_2} < m_{N_3} \simeq m_{N_4} < m_{N_5} \simeq m_{N_6}$. If the eigenvalues of each 3×3 matrix satisfy $\mu_s \ll m_{Di} \ll M_{Di}$ as was assumed in [8], the light mass eigenvalue is roughly given by $M_\nu \sim \mu_s (m_D/M_D)^2$. The MNS mixing matrix U_{MNS} is the first 3×3 part of this unitary matrix U,

$$U = \begin{pmatrix} & U_{eA} \\ U_{\mathrm{MNS}} & U_{\mu A} \\ & U_{\tau A} \\ * & * \end{pmatrix} . \tag{6}$$

Here the label A runs over the extra mass eigenstates $A = 4, \ldots, 9$, and the extraordinary matrix element U_{eA} gives a sterile to active neutrino mixing angle that have to be small enough so as to satisfy the current experimental bound, which is obtained from the invisible decays of the Z boson measured in L3 experiment at LEP.

After integrating out the heavy singlets, ν^c and S, we obtain the effective light neutrino mass matrix as

$$M_\nu = \left(M_D^{-1} m_D\right)^{\mathrm{T}} \mu_s \left(M_D^{-1} m_D\right) . \tag{7}$$

This light Majorana mass matrix can be diagonalised by the MNS matrix,

$$U_{\mathrm{MNS}}^{\mathrm{T}} M_\nu U_{\mathrm{MNS}} = \mathrm{diag}(m_1, m_2, m_3) . \tag{8}$$

An important fact is that the new physics scale has also the "see-saw structure" as

$$\Lambda \cong \frac{M_D^2}{\mu_s} . \tag{9}$$

Hence this mechanism is sometimes called as "double see-saw" mechanism. It's not the actual see-saw type but the inverse see-saw form, because the small lepton number violating ($\not{L}$) scale μ_s would indicate the large scale.

2 Fermion masses in an SO(10) model with a singlet

In order to make a prediction on the second Dirac neutrino mass matrix M_D, we need an information for the Yukawa couplings of Y_ν. In this paper, we make the minimal SO(10) model extend to add a number of singlet, which preserves a precise information for m_D.

Now we give a brief review of the minimal SUSY SO(10) model proposed in [17] and recently analysed in detail in [18–28]. Even when we concentrate our discussion on the issue of how to reproduce the realistic fermion mass matrices in the SO(10) model, there are lots of possibilities of the introduction of Higgs multiplets. The minimal supersymmetric SO(10) model includes only one **10** and one $\overline{\mathbf{126}}$ Higgs multiplets in Yukawa couplings with **16** matter multiplets. Here, in addition to it, we introduce a number of SO(10) singlet chiral superfields **1** as new matter multiplets.[1] This additional singlet can provide a double see-saw mechanism as described in the previous section. The relevant superpotential can be written as

$$\begin{aligned} W_Y = {} & Y_{10}^{ij} \mathbf{16}_i \mathbf{16}_j \mathbf{10}_H + Y_{126}^{ij} \mathbf{16}_i \mathbf{16}_j \overline{\mathbf{126}}_H \\ & + Y_s^{ij} \mathbf{16}_i \mathbf{1}_j \overline{\mathbf{16}}_H + \mu_s \mathbf{1}_i^2 . \end{aligned} \tag{10}$$

At low energy after the GUT symmetry breaking, the superpotential leads to

$$\begin{aligned} W = {} & \left(Y_{10}^{ij} H_{10}^u + Y_{126}^{ij} H_{126}^u\right) u_i^c q_j + \left(Y_{10}^{ij} H_{10}^d + Y_{126}^{ij} H_{126}^d\right) d_i^c q_j \\ & + \left(Y_{10}^{ij} H_{10}^u - 3 Y_{126}^{ij} H_{126}^u\right) N_i \ell_j \\ & + \left(Y_{10}^{ij} H_{10}^d - 3 Y_{126}^{ij} H_{126}^d\right) e_i^c \ell_j + Y_s^{ij} N_i S_j H_s + \mu_s S_i^2 , \end{aligned} \tag{11}$$

where H_{10} and H_{126} correspond to the Higgs doublets in $\mathbf{10}_H$ and $\overline{\mathbf{126}}_H$. That is, we have two pairs of Higgs doublets. In order to keep the successful gauge coupling unification, we suppose that one pair of Higgs doublets (a linear combination of $H_{10}^{u,d}$ and $H_{126}^{u,d}$) is light while the other pair

[1] The singlet matter multiplet may have it's origin in some E_6 representations **27** or **78** which are decomposed under the SO(10) subgroup as $\mathbf{27} = \mathbf{16} + \mathbf{10} + \mathbf{1}$, $\mathbf{78} = \mathbf{45} + \mathbf{16} + \overline{\mathbf{16}} + \mathbf{1}$. In such a case, the superpotential given in (10) may be generated from the following E_6 invariant superpotential: $W_Y = Y_1^{ij} \mathbf{27}_i \mathbf{27}_j \mathbf{27}_H + Y_2^{ij} \mathbf{27}_i \mathbf{27}_j \mathbf{351}'_H + Y_3^{ij} \mathbf{27}_i \mathbf{78}_j \overline{\mathbf{27}}_H + \mu^{ij} \mathbf{27}_i \mathbf{27}_j$.

is heavy ($\simeq M_{\rm GUT}$). The light Higgs doublets are identified as the MSSM Higgs doublets (H_u and H_d) and given by

$$H_u = \tilde{\alpha}_u H_{10}^u + \tilde{\beta}_u H_{126}^u \,; \quad H_d = \tilde{\alpha}_d H_{10}^d + \tilde{\beta}_d H_{126}^d \,, \quad (12)$$

where $\tilde{\alpha}_{u,d}$ and $\tilde{\beta}_{u,d}$ denote elements of the unitary matrix which rotate the flavour basis in the original model into the SUSY mass eigenstates. Omitting the heavy Higgs mass eigenstates, the low energy superpotential is described by only the light Higgs doublets H_u and H_d such that

$$\begin{aligned} W_Y = &\left(\alpha^u Y_{10}^{ij} + \beta^u Y_{126}^{ij}\right) u_i^c q_j H_u + \left(\alpha^d Y_{10}^{ij} + \beta^d Y_{126}^{ij}\right) d_i^c q_j H_d \\ &+ \left(\alpha^u Y_{10}^{ij} - 3\beta^u Y_{126}^{ij}\right) N_i \ell_j H_u \\ &+ \left(\alpha^d Y_{10}^{ij} - 3\beta^d Y_{126}^{ij}\right) e_i^c \ell_j H_d + Y_s^{ij} N_i S_j H_s + \mu_s S_i^2 \,, \end{aligned} \quad (13)$$

where the formulas of the inverse unitary transformation of (12), $H_{10}^{u,d} = \alpha^{u,d} H_{u,d} + \cdots$ and $H_{126}^{u,d} = \beta^{u,d} H_{u,d} + \cdots$, have been used. Providing the Higgs VEV's, $\langle H_u \rangle = v \sin\beta$ and $\langle H_d \rangle = v \cos\beta$ with $v \simeq 174\,\mathrm{GeV}$, the Dirac mass matrices can be read off as

$$\begin{aligned} M_u &= c_{10} M_{10} + c_{126} M_{126} \,, \\ M_d &= M_{10} + M_{126} \,, \\ m_D &= c_{10} M_{10} - 3 c_{126} M_{126} \,, \\ M_e &= M_{10} - 3 M_{126} \,, \end{aligned} \quad (14)$$

where M_u, M_d, m_D and M_e denote up-type quark, down-type quark, Dirac neutrino and charged-lepton mass matrices, respectively. Note that all the quark and lepton mass matrices are characterised by only two basic mass matrices, M_{10} and M_{126}, and four complex coefficients c_{10} and c_{126}. In addition to the above mass matrices the above model indicates the mass matrices,

$$\begin{aligned} M_R &= c_R M_{126} \,, \\ M_L &= c_L M_{126} \,, \end{aligned} \quad (15)$$

together with M_D given in (4). c_R and c_L correspond to the VEV's of $(\mathbf{10},\mathbf{1},\mathbf{3}) \subset \overline{\mathbf{126}}$ and $(\overline{\mathbf{10}},\mathbf{3},\mathbf{1}) \subset \overline{\mathbf{126}}$ under the the Pati–Salam subgroup, $G_{422} = \mathrm{SU}(4)_c \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$.

If M_R, M_L, M_D terms dominate, they are called Type-I, Type-II, and double see-saw, respectively. In this paper, we consider the case $c_R = c_L = 0$, double.

The mass matrix formulas in (14) leads to the GUT relation among the quark and lepton mass matrices,

$$M_e = c_d (M_d + \kappa M_u) \,, \quad (16)$$

where

$$c_d = -\frac{3c_{10} + c_{126}}{c_{10} - c_{126}} \,, \quad (17)$$

$$\kappa = -\frac{4}{3c_{10} + c_{126}} \,. \quad (18)$$

Without loss of generality, we can take the basis where M_u is real and diagonal, $M_u = D_u$. Since M_d is the symmetric matrix, it is described as $M_d = V_{\rm CKM}^* D_d V_{\rm CKM}^\dagger$ by using the CKM matrix $V_{\rm CKM}$ and the real diagonal mass matrix D_d. Considering the basis-independent quantities, $\mathrm{tr}[M_e^\dagger M_e]$, $\mathrm{tr}[(M_e^\dagger M_e)^2]$ and $\det[M_e^\dagger M_e]$, and eliminating $|c_d|$, we obtain two independent equations,

$$\left(\frac{\mathrm{tr}\left[\tilde{M}_e^\dagger \tilde{M}_e\right]}{m_e^2 + m_\mu^2 + m_\tau^2} \right)^2 = \frac{\mathrm{tr}\left[\left(\tilde{M}_e^\dagger \tilde{M}_e\right)^2\right]}{m_e^4 + m_\mu^4 + m_\tau^4} \,, \quad (19)$$

$$\left(\frac{\mathrm{tr}[\tilde{M}_e^\dagger \tilde{M}_e]}{m_e^2 + m_\mu^2 + m_\tau^2} \right)^3 = \frac{\det[\tilde{M}_e^\dagger \tilde{M}_e]}{m_e^2 m_\mu^2 m_\tau^2} \,, \quad (20)$$

where $\tilde{M}_e \equiv V_{\rm CKM}^* D_d V_{\rm CKM}^\dagger + \kappa D_u$. With input data of six quark masses, three angles and one CP-phase in the CKM matrix and three charged-lepton masses, we can solve the above equations and determine κ and $|c_d|$, but one parameter, the phase of c_d, is left undetermined [18–21]. With input data of six quark masses, three angles and one CP-phase in the CKM matrix and three charged lepton masses, we solve the above equations and determine κ. The original basic mass matrices, M_{10} and M_{126}, are described by

$$M_{10} = \frac{3 + |c_d| \mathrm{e}^{\mathrm{i}\sigma}}{4} V_{\rm CKM}^* D_d V_{\rm CKM}^\dagger + \frac{|c_d| \mathrm{e}^{\mathrm{i}\sigma} \kappa}{4} D_u \,, \quad (21)$$

$$M_{126} = \frac{1 - |c_d| \mathrm{e}^{\mathrm{i}\sigma}}{4} V_{\rm CKM}^* D_d V_{\rm CKM}^\dagger - \frac{|c_d| \mathrm{e}^{\mathrm{i}\sigma} \kappa}{4} D_u \,, \quad (22)$$

as the functions of σ, the phase of c_d, with the solutions $|c_d|$ and κ determined by the GUT relation.

Now let us solve the GUT relation and determine $|c_d|$ and κ. Since the GUT relation of (16) is valid only at the GUT scale, we first evolve the data at the weak scale to the corresponding quantities at the GUT scale with given $\tan\beta$ according to the renormalization group equations (RGE's) and use them as input data at the GUT scale. Note that it is non-trivial to find the solution of the GUT relation since the number of the free parameters (fourteen) is almost the same as the number of inputs (thirteen). The solution of the GUT relation exists only if we take appropriate input parameters. Taking the experimental data at the M_Z scale [31], we get the following values for charged fermion masses and the CKM matrix at the GUT scale, $M_{\rm GUT}$ with $\tan\beta = 10$:

$$\begin{aligned} m_u &= 0.000980 \,, & m_c &= 0.285 \,, & m_t &= 113 \,, \\ m_d &= 0.00135 \,, & m_s &= 0.0201 \,, & m_b &= 0.996 \,, \\ m_e &= 0.000326 \,, & m_\mu &= 0.0687 \,, & m_\tau &= 1.17 \,, \end{aligned}$$

and

$$V_{\rm CKM}(M_{\rm GUT}) = \begin{pmatrix} 0.975 & 0.222 & -0.000940 - 0.00289\mathrm{i} \\ -0.222 - 0.000129\mathrm{i} & 0.974 + 0.000124\mathrm{i} & 0.0347 \\ 0.00864 - 0.00282\mathrm{i} & -0.0337 - 0.000647\mathrm{i} & 0.999 \end{pmatrix}$$

in the standard parameterisation. The signs of the input fermion masses have been chosen to be $(m_u, m_c, m_t) = (+,-,+)$ and $(m_d, m_s, m_b) = (-,-,+)$. By using these

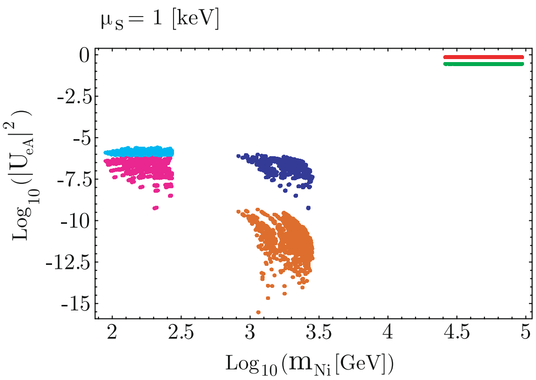

Fig. 1. Constraint on the heavy sterile mixing angles in the cases of $\mu_s = 1\,\mathrm{keV}$ with φ_1, φ_2 and σ varied from 0 to 2π

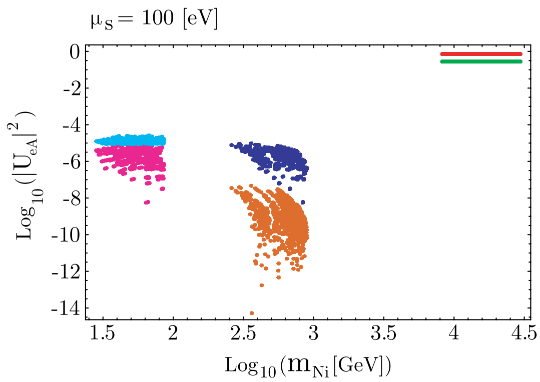

Fig. 2. Constraint on the heavy sterile mixing angles in the cases of $\mu_s = 100\,\mathrm{eV}$ with φ_1, φ_2 and σ varied from 0 to 2π

outputs at the GUT scale as input parameters, we can solve (19) and (20) and find a solution:

$$\kappa = -0.0103 + 0.000606\mathrm{i}\,, \qquad |c_d| = 6.32\,. \tag{23}$$

Once these parameters, $|c_d|$ and κ, are determined, we can describe all the fermion mass matrices as a functions of σ from the mass matrix formulas of (14), (21) and (22). Thus in the minimal SO(10) model we have almost unambiguous Dirac neutrino mass matrix m_D and, therefore, we can obtain the informations on M_D from the neutrino experiments via $M_\nu = (M_D^{-1} m_D)^{\mathrm{T}} \mu_s (M_D^{-1} m_D)$ as in (7).

Now we proceed to the numerical calculation of M_D from the well-confirmed neutrino oscillation data. The MNS mixing matrix U in the standard parametrization is

$$U = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12}\mathrm{e}^{\mathrm{i}\varphi_2} & s_{13}\mathrm{e}^{\mathrm{i}(\varphi_1-\delta)} \\ (-c_{23}s_{12} - s_{23}c_{12}s_{13}\mathrm{e}^{\mathrm{i}\delta})\mathrm{e}^{-\mathrm{i}\varphi_2} & c_{23}c_{12} - s_{23}s_{12}s_{13}\mathrm{e}^{\mathrm{i}\delta} & s_{23}c_{13}\mathrm{e}^{\mathrm{i}(\varphi_1-\varphi_2)} \\ (s_{23}s_{12} - c_{23}c_{12}s_{13}\mathrm{e}^{\mathrm{i}\delta})\mathrm{e}^{-\mathrm{i}\varphi_1} & (-s_{23}c_{12} - c_{23}s_{12}s_{13}\mathrm{e}^{\mathrm{i}\delta})\mathrm{e}^{-\mathrm{i}(\varphi_1-\varphi_2)} & c_{23}c_{13} \end{pmatrix}, \tag{24}$$

where $s_{ij} := \sin\theta_{ij}$, $c_{ij} := \cos\theta_{ij}$ and δ, φ_1, φ_2 are the Dirac phase and the Majorana phases [29, 30], respectively. Recent KamLAND data tells us that[2]

$$\begin{aligned} \Delta m_\oplus^2 &= \Delta m_{32}^2 = 2.1 \times 10^{-3}\,\mathrm{eV}^2\,, \\ \sin^2\theta_\oplus &= 0.5\,, \\ \Delta m_\odot^2 &= \left|\Delta m_{21}^2\right| = 8.3 \times 10^{-5}\,\mathrm{eV}^2\,, \\ \sin^2\theta_\odot &= 0.28\,, \\ |U_{e3}|^2 &< 0.061\,. \end{aligned} \tag{25}$$

For simplicity we take $U_{e3} = 0$. Note that we can take both signs of Δm_{21}^2, $\Delta m_{21}^2 > 0$ or $\Delta m_{21}^2 < 0$. The former is called normal hierarchy, the latter is called inverted hierarchy. Here we adopt the former case, and take the lightest neutrino mass eigenvalue as $m_\ell = 10^{-3}$ eV. Then the mass eigenvalues are written as

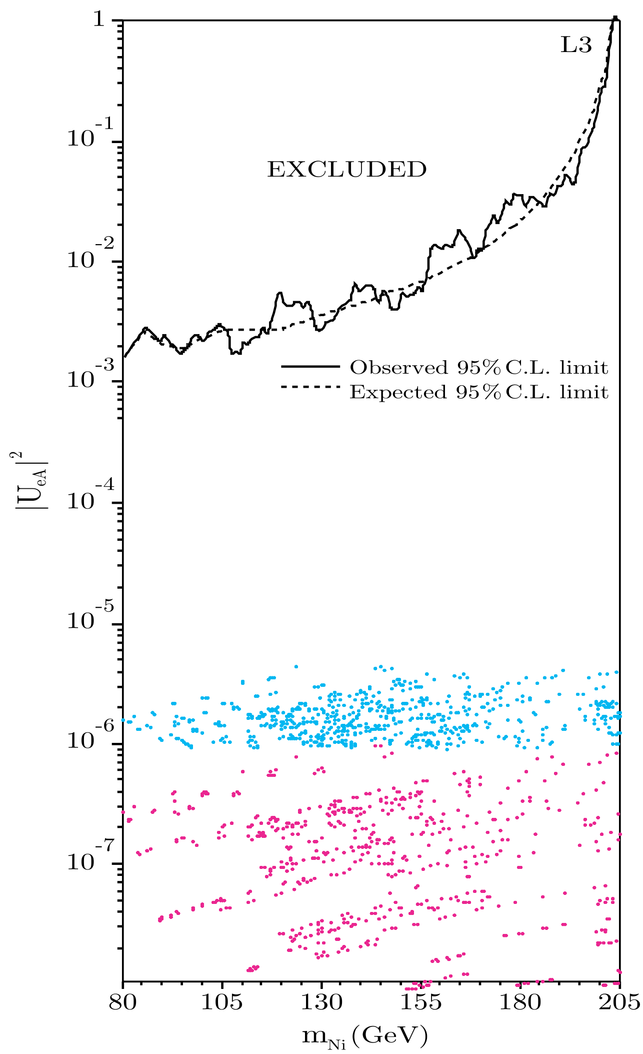

Fig. 3. The same as Fig. 1, but with the LEP experimental bound superimposed

$$m_1 = m_\ell\,,$$

[2] Our convention is $\Delta m_{ij}^2 = m_i^2 - m_j^2$.

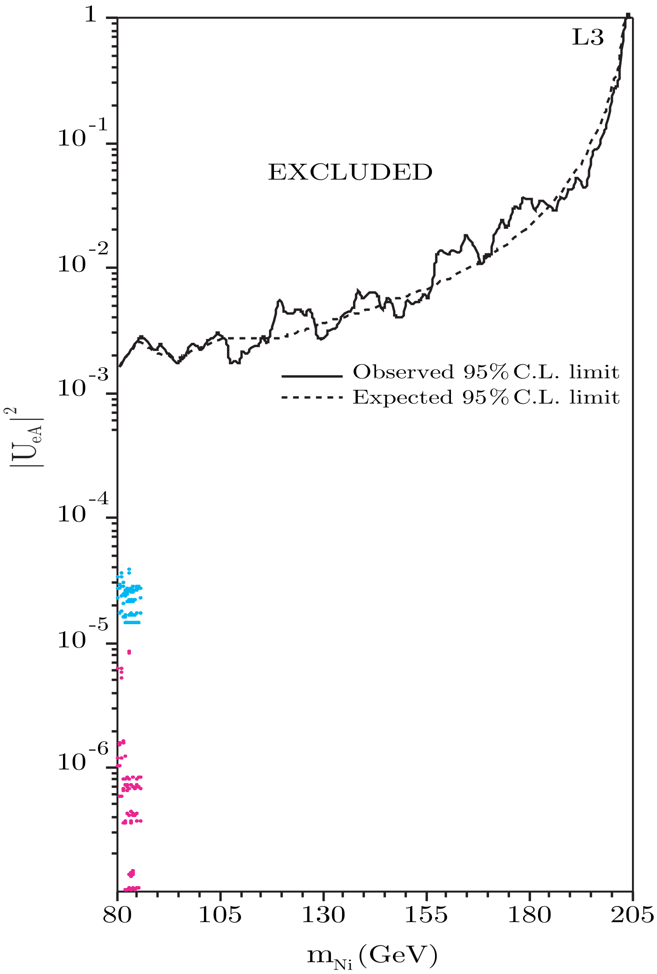

Fig. 4. The same as Fig. 2, but with the LEP experimental bound superimposed

$$m_2 = \sqrt{m_\ell^2 + \Delta m_\oplus^2}\,,$$
$$m_3 = \sqrt{m_\ell^2 + \Delta m_\oplus^2 + \Delta m_\odot^2}\,. \tag{26}$$

For the light Dirac neutrino mass matrix m_D, we input the SO(10) predicted one as was done in the previous section. However, unlike the case of minimal SO(10) GUT model, we can not fix σ. (the only unknown parameter in the minimal SO(10) model before fitting with neutrino oscillation data [18]). So we can obtain the heavy Dirac neutrino mass matrix M_D as a function of μ_s and the three undetermined parameters, σ, two Majorana phases φ_1 and φ_2 in the MNS mixng matrix for fixed $U_{e3} = 0$. We note that the Dirac phase has little effect on our calculations if U_{e3} has non-zero tiny values.

Then, we get a prediction on the mass spectra and the active to sterile neutrino mixing angles for $\mu_s = 1\,\text{keV}$ in Figs. 1 and 2. In these figures we varied the parameters φ_1, φ_2 and σ from 0 to 2π. The same results for the case of $\mu_s = 100\,\text{eV}$ are shown in Figs. 3 and 4. This shows that if the parameter μ_s varies from 1 keV to 100 eV, then we obtain the result which shows one order of magnitude larger mixing angles and one half order of magnitude smaller mass eigenvalues. That is similar for the case of larger value of the parameter μ_s. These results of Figs. 3 and 4 show that there exist a parameter space, which is allowed by the LEP experimental bound [14]. The allowed ranges for each mass eigenvalues and the mixing angles are listed in Tables 1 and 2.

Also it may be worthwhile noticing that such keV scale lepton number violation may lead to an interesting signature in the neutrinoless double beta decay [15] or becomes a possible candidate for the cold dark matter [16]. These subjects are the topics for the future study.

Table 1. The allowed ranges for the mass eigenvalues and mixing angles in the case of $\mu_s = 1\,\text{keV}$

The allowed ranges for mass eigenvalues	The allowed ranges for mixing angles
89.332 GeV $< m_{N_1} <$ 270.13 GeV	$-6.0632 < \log_{10}(\lvert U_{e4}\rvert^2) < -5.5840$
89.332 GeV $< m_{N_2} <$ 270.13 GeV	$-9.2564 < \log_{10}(\lvert U_{e5}\rvert^2) < -6.0151$
819.72 GeV $< m_{N_3} <$ 2.8259 TeV	$-9.2564 < \log_{10}(\lvert U_{e6}\rvert^2) < -6.0151$
819.72 GeV $< m_{N_4} <$ 2.8259 TeV	$-15.530 < \log_{10}(\lvert U_{e7}\rvert^2) < -9.3336$
25.988 TeV $< m_{N_5} <$ 93.410 TeV	$-0.55515 < \log_{10}(\lvert U_{e8}\rvert^2) < -0.55064$
25.988 TeV $< m_{N_6} <$ 93.410 TeV	$-0.14498 < \log_{10}(\lvert U_{e9}\rvert^2) < -0.14047$

Table 2. The same as Table 1, but in the case of $\mu_s = 100\,\text{eV}$

The allowed ranges for mass eigenvalues	The allowed ranges for mixing angles
28.254 GeV $< m_{N_1} <$ 85.443 GeV	$-5.0632 < \log_{10}(\lvert U_{e4}\rvert^2) < -4.5841$
28.254 GeV $< m_{N_2} <$ 85.443 GeV	$-8.2575 < \log_{10}(\lvert U_{e5}\rvert^2) < -5.0161$
259.26 GeV $< m_{N_3} <$ 893.68 GeV	$-8.2575 < \log_{10}(\lvert U_{e6}\rvert^2) < -5.0161$
259.26 GeV $< m_{N_4} <$ 893.68 GeV	$-14.284 < \log_{10}(\lvert U_{e7}\rvert^2) < -7.3339$
8.2182 TeV $< m_{N_5} <$ 29.540 TeV	$-0.55294 < \log_{10}(\lvert U_{e8}\rvert^2) < -0.55285$
8.2182 TeV $< m_{N_6} <$ 29.540 TeV	$-0.14268 < \log_{10}(\lvert U_{e9}\rvert^2) < -0.14267$

Finally, it is remarkable to say that the see-saw mechanism itself (or the types of it) can never been proofed and all the models should take care of all the types of the see-saw mechanism including the alternatives to it [32, 33]. The test of all these models is due to the applications to the other phenomelogical consequences, for example, the lepton flavour violating processes and so on [34, 35].

3 Summary

In this paper, we have constructed an SO(10) model in which the smallness of the neutrino masses are explained in terms of the double see-saw mechanism. To evaluate the parameters related to the singlet neutrinos, we have used the minimal SUSY SO(10) model. This model can simultaneously accommodate all the observed quark-lepton mass matrix data with appropriately fixed free parameters. Especially, the neutrino-Dirac–Yukawa coupling matrix are completely determined. Using this Yukawa coupling matrix, we have calculated the masses and mixings for the not-so-heavy singlet neutrinos. The obtained ranges of the mass of M_D is interesting since they are potentially testable by a forthcoming LHC experiment.

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